Linear Programming

Finite Math

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General Description of Linear Programming

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In a *linear programming problem*, we are concerned with *optimizing* (finding the maximum and minimum values, called the *optimal values*) of a linear *objective function* z of the form

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where a and b are not both zero and the *decision variables* x and y are subject to *constraints* given by linear inequalities.

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$$z = ax + by$$

where a and b are not both zero and the *decision variables* x and y are subject to *constraints* given by linear inequalities.Additionally, x and y must be nonnegative, i.e., $x \geq 0$ and $y \geq 0$.

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Theorem (Fundamental Theorem of Linear Programming)

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(B) *If the feasible region is unbounded and the coefficients of the objective function are positive, then the minimum value of the objective function exists, but the maximum value does not.*

(C) *If the feasible region is empty, then both the maximum value and the minimum value of the objective function do not exist.*

Geometric Method for Solving Linear Programming Problems

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2. *Construct a corner point table listing the value of the objective function at each corner point.*
3. *Determine the optimal solution(s) from the table in Step 2 (smallest=minimum, largest=maximum).*
4. *For an applied problem, interpret the optimal solution(s) in terms of the original problem.*

Now You Try It!

Example

Maximize and minimize $z = 2x + 3y$ *subject to*

$$\begin{aligned} 2x + y &\geq 10 \\ x + 2y &\geq 8 \\ x, y &\geq 0 \end{aligned}$$

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Solution

Minimum of $z = 14$ at $(4, 2)$. No maximum.

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Example

Maximize and minimize $P = 30x + 10y$ *subject to*

$$\begin{aligned} 2x + 2y &\geq 4 \\ 6x + 4y &\leq 36 \\ 2x + y &\leq 10 \\ x, y &\geq 0 \end{aligned}$$

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$$\begin{aligned} 2x + 2y &\geq 4 \\ 6x + 4y &\leq 36 \\ 2x + y &\leq 10 \\ x, y &\geq 0 \end{aligned}$$

Solution

Minimum of $P = 20$ *at* $(0, 2)$. *Maximum of* $P = 150$ *at* $(5, 0)$.

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Example

Maximize and minimize $P = 3x + 5y$ *subject to*

$$\begin{aligned} x + 2y &\leq 6 \\ x + y &\leq 4 \\ 2x + 3y &\geq 12 \\ x, y &\geq 0 \end{aligned}$$

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Example

Maximize and minimize $P = 3x + 5y$ *subject to*

$$\begin{aligned} x + 2y &\leq 6 \\ x + y &\leq 4 \\ 2x + 3y &\geq 12 \\ x, y &\geq 0 \end{aligned}$$

Solution

No optimal solutions.